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Electric Currents in the Ionosphere. III. Ionization Drift due to Winds and Electric Fields

D. F. Martyn

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ELECTRIC CURRENTS IN THE IONOSPHERE

III. IONIZATION DRIFT DUE TO WINDS AND ELECTRIC FIELDS

By D. F. MARTYN, F.R.S.

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CONTENTS

An analysis is made of the drift velocity of the (neutral) ionization in a uniform ionosphere under the influences of an electric field and/or atmospheric wind. It is shown that this drift of ionization produces the Ampere body force on the medium; the electric current flows perpendicular to the drift.

The motion of a cylinder of ionization, of density differing from the surrounding medium, is then studied. It is found that the motion is electrodynamically stable, but unstable hydrodynamically, if Hall conductivity is appreciable. In the latter event there is rapid accretion of (neutral) ionization on one side of the cylinder, depletion on the other. It is suggested that this is the origin of sporadic E (E_s) ionization, and is likely to be an important factor in the production of the long-enduring meteor trails detected by radio methods.

Formulae are derived for the horizontal and vertical drift of ionization at all latitudes in a thin ionosphere in which vertical electric currents are prohibited by polarization. Graphs are given which permit derivation of the true wind or field in a given region of the ionosphere from experimental observations of the drift velocities.

1. Introduction

In part I the conductivity of the ionosphere, regarded as a thin conducting sheet, has been deduced at all heights and geomagnetic latitudes, for north-south and for east-west electric fields. In part II the 'atmospheric dynamo' theory of Stewart and Schuster has been applied to deduce the currents flowing in the ionosphere under the influence of tidal winds in that region. In the present paper it is proposed to discuss the mass motion of the ionization in the ionosphere under the influence of winds and electric fields. Such observations as are possible on these motions necessarily refer to the movements of patches or 'blobs' of ionization whose density differs in greater or lesser degree from that of their surroundings. One object of this paper is to relate the velocity of such 'blobs' to that of the local wind and/or to the field in the medium, for all regions of the ionosphere. Without knowledge of these relationships interpretation of the radio observations in terms of 'winds' is impossible.

In the course of the investigation it has been found that the motion of ionization, at levels where the Hall conductivity is important, is electrodynamically stable, but hydrodynamically unstable. A small 'blob' of ionization tends to move in such a way that the density is greatly increased over part of its surface, and diminished over another part. It is probable that this hydrodynamic instability, which necessarily will be specially important in the E region, is the fundamental cause of sporadic E ionization. The latter (E_s) has been found by some workers to have the characteristics of 'blobs', by others to behave like thin sheets of unusual electron density. According to the results of the present investigation a 'blob' of slightly unusual density, such (for example) as might be produced by the residual ionized matter of a meteor, would rapidly acquire on its surface a thin sheet of markedly unusual density, thus acquiring both of the characteristics attributed by experimenters to E_s. It seems likely also that this phenomenon has a significant bearing on the anomalously long duration meteor trails observed by radio methods; the decay of such trails by diffusion and recombination will be opposed by accretion on their surfaces.

2. Ionization drift in a uniform medium

The movement of ionized air parallel to a magnetic field H is uninfluenced by the field. It will be sufficient therefore to consider electric fields and motions perpendicular to H.

Take cylindrical polar co-ordinates such that H is anti-parallel to the z axis. All winds and fields are assumed to lie in the (r, ϕ) plane. It will suffice, for a uniform medium, to consider the case where E, the electric field, and c_0 the wind speed, are uniform vectors, so that their components in the r, ϕ directions are functions of ϕ only. The motion of the ionization may be deduced by calculating the mass motion of ionization arising from the two fields, E, and the 'dynamo' field $c_0 H$, and superposing c_0 upon the resultant. Thus in general it is sufficient to consider the mass motion of ionization due to a field E, remembering that if this has a component due to local 'dynamo' action then the local wind producing this 'dynamo' field must be superposed on the resulting motion of ionization.

Suppose, without loss of generality, that E is directed along the x axis. Then

$$E_r = E\cos\phi, \quad E_\phi = -E\sin\phi. \tag{1}$$

The currents $j_{r,\phi}$ per cm^2 are then given by

$$\left.\begin{aligned} j_r &= \sigma_1 E_r + \sigma_2 E_\phi, \\ j_\phi &= \sigma_1 E_\phi - \sigma_2 E_r, \end{aligned}\right\} \tag{2}$$

using the notation given in I for the Pedersen and Hall conductivities σ_1 and σ_2 respectively.

Since we are concerned only with slow changes of ionization density, for which the period is much greater than the plasma periods, it follows that the medium is substantially electrically neutral, so that $N_e = N_i = N$. Hence

$$\begin{aligned} j_r &= -Nec_e + Nec_i \\ &= Ne(\sin 2\alpha_e + \sin 2\alpha_i)\, E_r/2H + Ne(\sin^2\alpha_e - \sin^2\alpha_i)\, E_\phi/H, \end{aligned} \tag{3}$$

using the notation of I for $\alpha_{e,i}$ and putting $c_{e,i}$ for the radial electron, ion velocities. Then

$$\left.\begin{aligned} c_e &= -\sin\alpha_e \cos\alpha_e E_r/H - \sin^2\alpha_e E_\phi/H \\ &= -\sin\alpha_e \cos(\phi+\alpha_e)\, E/H, \end{aligned}\right\} \tag{4}$$

$$c_i = \sin\alpha_i \cos(\phi - \alpha_i)\, E/H. \tag{5}$$

The direction ϕ_0 for which $c_e = c_i$ is given by

$$\tan\phi_0 = \cot(\alpha_e - \alpha_i)$$

or

$$\phi_0 = \tfrac{1}{2}\pi - \alpha_e + \alpha_i.$$

In this direction

$$c_{0e} = c_{0i} = \sin\alpha_e \sin\alpha_i E/H. \tag{6}$$

ϕ_0 and c_{0e} define the velocity of 'neutral' ionization drift.

Since the motions are rectilinear the direction ϕ_j in which c_e and c_i are opposed, will be perpendicular to ϕ_0; this is the direction of the electric current, whose sense will be that of c_i, thus lying in the fourth quadrant, i.e. $\phi_j = (\alpha_i - \alpha_e)$.

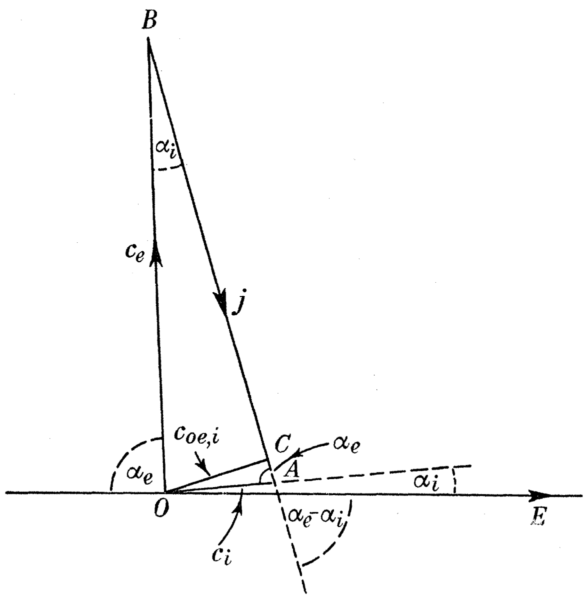

FIGURE 1. Ion and electron drift-velocity vectors in a uniform medium due to an electric field E. (The magnetic field is supposed perpendicular and inwards to the plane of the diagram.)

In this direction

$$\left.\begin{aligned} c_{je} &= -\sin\alpha_e \cos\alpha_i E/H, \\ c_{ji} &= \sin\alpha_i \cos\alpha_e E/H, \end{aligned}\right\} \tag{7}$$

and the current density is

$$j = Ne(c_{ji} - c_{je}) = Ne\sin(\alpha_e + \alpha_i)\, E/H.$$

The relations between the various vectors are shown in figure 1. Here the ion (electron) velocities are OA, (OB). OC is the vector drift velocity of the ionization and BCA the direction of the current. E is directed along the x-axis.

On the simplest kinetic considerations, the drift of ionization through the medium will produce a mechanical force F per cm^3, along the direction AC, of magnitude

$$F = N(m_e \nu_e + m_i \nu_i)\, c_{0e}.$$

Substituting c_{0e} from equation (6), and remembering (see I) that $\tan\alpha = \omega/\nu$, gives

$$F = ENe\sin(\alpha_e + \alpha_i). \tag{8}$$

According to the classical principles of electrodynamics the mechanical force F' per cm^3 produced by interaction of j and H is

$$F' = jH = ENe\sin(\alpha_e + \alpha_i) = F. \tag{9}$$

Thus the body force on the gas is seen to be due to the impact upon it of the ionization drifting at right-angles to the current; in the absence of this drift AOB would lie on a straight line and there could be no mechanical force on the medium.

It is of interest to examine the resultant mechanical force along the direction of the current. This is

$$N(m_i \nu_i c_{ji} + m_e \nu_e c_{je}) = NeE(\cos\alpha_i \cos\alpha_e - \cos\alpha_i \cos\alpha_e) = 0,$$

showing, in agreement with electrodynamical principles, that there is no mechanical force in the direction of the current.

Thus simple mechanical considerations show the impossibility of explaining the Hall effect by means of a single carrier; two carriers of opposite charge are necessary so that the mechanical forces in the direction of current flow may be balanced out.

Equation (6) shows that the drift velocity of the ionization is always less than E/H. In the lower regions of the ionosphere (the D and lower E regions) $\sin\alpha_i \ll 1$ and the drift is very small. In the F_2 region both α_e and α_i tend to $\frac{1}{2}\pi$, and the drift velocity nearly attains the maximum possible velocity (E/H); it is then directed nearly along the y-axis (figure 1). It must be recalled, however, that these remarks apply to the ionosphere only at the magnetic poles, where H is vertical.

If E is the 'dynamo field' due to an atmospheric wind, the velocity of the latter will be $-E/H$, directed along the y-axis. This velocity must then be superposed upon c_{0e} to give the resulting ionization drift. It follows that in the D and lower E regions, at the magnetic poles, the ionization will drift horizontally, substantially with the velocity of the local wind. In the F_2 region the motion of ionization due to local wind will be negligible, since the drift due to the 'dynamo' field is almost equal but opposite in direction to the wind velocity. It may safely be concluded therefore that if F_2 ionization be observed to move horizontally with high velocity near the magnetic poles then this motion is *not* due to local atmospheric wind, but is caused by an applied electric field.

In intermediate regions (E_2 and F_1) the drift due to the dynamo field will be inclined to the wind direction, and the resultant velocity of the ionization may be substantially different in direction and magnitude from that of the wind.

3. Motion of a uniformly ionized cylinder in a uniform medium of different ionization density

In the previous section we deduced the motion of ionization in a uniform medium under the influence of electric fields or local winds. It has not yet proved possible to detect movement of uniform ionization in the ionosphere. Experimental methods so far devised give the velocities of irregular patches or clouds of ionization, whose densities differ from that of their surroundings. It is known from the work of Chapman & Ferraro (1931) that a semi-infinite (neutral) cylinder or slab of ionized particles can move freely at right-angles to a magnetic field if immersed in a non-conducting medium; the walls of the cylinder or slab become polarized in such a way that the retarding influence of the magnetic field is neutralized inside the slab or cylinder. (The motion of such a slab or cylinder parallel or anti-parallel to the magnetic field is of course also unaffected.) It may safely be anticipated therefore that a dense column of ionization, produced (for example) by a meteorite in the poorly conducting lower D region, will move with the velocity of the local atmospheric wind. It is not at all clear, however, what will be the motion of such a column in the more highly conducting regions of the ionosphere. The polarization on its walls will necessarily leak away through the surrounding conductive medium, and the resulting transverse electric current within the cylinder must alter its motion. In this section we examine the effect of this leakage current. The conditions are similar to those of § 2, save that a vertically infinite cylinder of ionization of radius R and density N is centred on the origin of axes. For radial distances $r > R$ the ionization density is now taken as N'. As $N \to N'$ the problem reduces to that discussed in § 2.

The presence of this cylinder of conductivity differing from its surroundings will immediately modify the field in its vicinity in such a way that the current remains divergence-less. This will be achieved by the setting up of a polarization field of potential $S(r, \phi)$. Now

$$E_r = E\cos\phi - \partial S/\partial r, \tag{10}$$

$$E_\phi = -E\sin\phi - \partial S/r\,\partial\phi, \tag{11}$$

$$j_r = \sigma_1(E\cos\phi - \partial S/\partial r) - \sigma_2(E\sin\phi + \partial S/r\,\partial\phi), \tag{12}$$

$$-j_\phi = \sigma_1(E\sin\phi + \partial S/\partial\phi) + \sigma_2(E\cos\phi - \partial S/\partial r). \tag{13}$$

Eliminating j from (12) and (13), using the condition div $(j) = 0$, gives

$$\partial^2 S/\partial r^2 + \partial S/r\,\partial r + \partial^2 S/r^2\,\partial\phi^2 = 0. \tag{14}$$

This is a well-known equation, related to Bessel's, the appropriate solution being

$$S = QEr\cos(\phi+\eta) \tag{15}$$

within the cylinder, and

$$S' = QE\frac{R^2}{r}\cos(\phi+\eta') \tag{16}$$

in the surrounding medium. On the cylinder walls, $(r = R)$, we must have $S = S'$, so $\eta = \eta'$. Also, since there can be no continuing accretion of charge on the boundary walls, $j'_R = j_R$ for all ϕ's. These conditions give, using dashed superscripts for the parameters outside the cylinder,

$$Q^2 = \{(\sigma_1-\sigma'_1)^2 + (\sigma_2-\sigma'_2)^2\}/\{(\sigma_1+\sigma'_1)^2 + (\sigma_2-\sigma'_2)^2\}, \tag{17}$$

$$\tan\eta = 2\sigma'_1(\sigma_2-\sigma'_2)/\{\sigma_1^2 - \sigma_1'^2 + (\sigma_2-\sigma'_2)^2\}. \tag{18}$$

Write σ for $(\sigma_1^2+\sigma_2^2)^{\frac{1}{2}}$, and correspondingly for σ'; then

$$j_r = E\sigma(1+Q^2-2Q\cos\eta)^{\frac{1}{2}}\cos(\phi+\psi), \tag{19}$$

$$j_\phi = -E\sigma(1+Q^2-2Q\cos\eta)^{\frac{1}{2}}\sin(\phi+\psi), \tag{20}$$

$$j'_r = E\left[\sigma'^2\left(1+Q^2\frac{R^4}{r^4}\right) - 2Q\frac{R^2}{r^2}\{(\sigma_2'^2-\sigma_1'^2)\cos\eta - 2\sigma'_1\sigma'_2\sin\eta\}\right]^{\frac{1}{2}}\cos(\phi+\lambda), \tag{21}$$

$$j_\phi = -E\left[\sigma'^2\left(1+Q^2\frac{R^4}{r^4}\right) + 2Q\frac{R^2}{r^2}\{(\sigma_2'^2-\sigma_1'^2)\cos\eta - 2\sigma'_1\sigma'_2\sin\eta\}\right]^{\frac{1}{2}}\sin(\phi+\mu), \tag{22}$$

$$\tan\psi = \{\sigma_2(1-Q\cos\eta) - \sigma_1 Q\sin\eta\}/\{\sigma_1(1-Q\cos\eta) + \sigma_2 Q\sin\eta\}, \tag{23}$$

$$\tan\lambda = \left\{\sigma'_2\left(1-Q\frac{R^2}{r^2}\cos\eta\right) + \sigma'_1 Q\frac{R^2}{r^2}\sin\eta\right\}\bigg/\left\{\sigma'_1\left(1+Q\frac{R^2}{r^2}\cos\eta\right) + \left(\sigma'_2 Q\frac{R^2}{r^2}\sin\eta\right)\right\}, \tag{24}$$

$$\tan\mu = \left\{\sigma'_2\left(1+Q\frac{R^2}{r^2}\cos\eta\right) - \sigma'_1 Q\frac{R^2}{r^2}\sin\eta\right\}\bigg/\left\{\sigma'_1\left(1-Q\frac{R^2}{r^2}\cos\eta\right) - \sigma'_2 Q\frac{R^2}{r^2}\sin\eta\right\}. \tag{25}$$

It is sufficiently accurate, even for the F_2 region, to assume $\sigma'/\sigma = N'/N = k$, say. (In a fully-ionized medium and weak magnetic field σ is independent of N, since the collisional frequency of the charged particles then becomes proportional N.) On this assumption, permissible for the ionosphere, though not for cosmic problems, we have

$$Q = \sigma(1-k)/\{(1+k)^2\sigma^2 - 4k\sigma_2^2\}^{\frac{1}{2}} = \sigma(1-k)/a, \tag{26}$$

$$\tan\psi = \tan\eta = 2k\sigma_1\sigma_2/\{(1+k)\sigma^2 - 2k\sigma_2^2\} = \tan\lambda_R, \tag{27}$$

$$\tan\mu = \sigma_2\{a^2r^2 + R^2\sigma^2(1-k)^2\}/\sigma_1\{a^2r^2 - R^2\sigma^2(1-k^2)\}, \tag{28}$$

where
$$a = \{(1+k)^2\sigma^2 - 4k\sigma_2^2\}^{\frac{1}{2}}. \tag{29}$$

At great distances from the origin ($r\to\infty$)

$$j'_r = E\sigma' \cos\{\phi + \tan^{-1}(\sigma'_2/\sigma'_1)\}, \tag{30}$$

in agreement with the current density derived for a uniform medium (j_r in equation (2)). It is readily verified also that for $k = 1$ (a completely uniform medium) $Q = 0$, and equation (30) gives the radial current density in all regions, the tangential density being

$$j_\phi = -E\sigma\sin\{\phi + \tan^{-1}(\sigma_2/\sigma_1)\}. \tag{31}$$

When the medium outside the cylinder is non-conducting $k = \sigma' = \eta = 0$, $Q = 1$, and there are no currents anywhere; the cylinder is completely polarized.

Write $U_{r,\phi}$ for the velocities of the ionization through the gas. This is the drift motion which produces the mechanical force on the medium. It must be at right angles to the current, and given in magnitude (equations (6), (8), (9)) by

$$U_r = -Gj_\phi, \tag{32}$$

$$U_\phi = Gj_r \tag{33}$$

where G is $\tan\alpha_e \tan\alpha_i/H\sigma_0$, σ_0 being the conductivity *parallel* to the field H (i.e.

$$\sigma_0 = Ne\sin(\alpha_e+\alpha_i)/H\cos\alpha_e\cos\alpha_i,$$

cf. I).

At the cylinder boundary
$$\left.\begin{aligned} U_R &= \frac{GE\sigma^2}{\sigma_2}\sin\eta\sin(\phi+\eta) \\ &= \frac{2kGE\sigma\sigma_1}{a}\sin(\phi+\eta), \end{aligned}\right\} \tag{34}$$

$$U_{\phi(R)} = \frac{GE\sigma^2}{\sigma_2}\sin\eta\cos(\phi+\eta), \tag{35}$$

$$U'_R = \frac{GE\sigma^2}{\sigma_2}\sin\eta\left\{1 + \frac{(1-k)^2\sigma_2^2}{k^2\sigma_1^2}\right\}^{\frac{1}{2}}\sin(\phi+\mu_R), \tag{36}$$

$$U'_{\phi(R)} = G'j'_R = Gj_R/k = \frac{GE\sigma^2}{k\sigma_2}\sin\eta\cos(\phi+\eta), \tag{37}$$

$$\tan\mu_R = \sigma_2\left(k + \frac{1}{k} - 2\frac{\sigma_2^2}{\sigma^2}\right)\Big/\sigma_1\left(k + 1 - 2\frac{\sigma_2^2}{\sigma^2}\right). \tag{38}$$

It is readily verified that $\operatorname{div}(U) = 0$, both inside and outside the cylinder. However, $U_R \neq U'_R$, either in amplitude or direction, so that there must be continuous accumulation of (neutral) ionization on the walls of the cylinder in certain parts, and withdrawal from the wall in others. The motion of the uniform cylinder, although satisfying the laws of electrodynamics, cannot satisfy those of hydrodynamics without density changes occurring at the boundary. Thus if a limited region of the ionosphere differs *slightly* in ionization density from its surroundings we may expect notable gradients of ionization to appear on its surface. It seems probable that this effect is the fundamental cause of so-called sporadic E (E_s) ionization, which is known to behave partly like 'clouds' of ionization of density differing from the surroundings, and partly like thin sheets of unusual density. The instability vanishes as $\sigma_2 \to 0$, since then $U_R \to U'_R$; thus we may expect it to be relatively absent in the F_2 region, and specially prominent in the E region, where σ_2/σ_1 is greatest.

It is clear that, in the ionosphere, motion of the cylinder like a solid body, with velocity U_R, is not possible, since its walls will be deformed. On the dubious assumption that the accretion (or depletion) of ionization on the walls is immediately diffused uniformly through the cylinder the problem becomes tractable. If u_R is $U_R - U'_R$ the surface would have a component of motion ku_R and the resultant motion 'as a solid' would be obtained by combining this with U_R. In the ionosphere at the level of the E region diffusion is slow, and there is no chance of this approximation being valid. Instead, it seems certain that the cylinder will be deformed, eventually breaking up into two regions, one of abnormally high and one of abnormally low density, which will drift apart. A full discussion of this problem would be difficult, and cannot be attempted here. It must suffice to point out that the existence of an electric field (or wind) at ionospheric levels near the E region must cause unstable motion of the ionization, such that small irregularities of distribution are enhanced, each patch of small irregularity first developing two thin regions of high and low density on opposite surfaces, and eventually probably breaking up into smaller patches of enhanced irregularity in density.

At great distances from the cylinder the radial velocity of the medium is (by equations (31), (32))

$$\begin{aligned} u'_r &= -G'j'_\phi = G'E\sigma' \sin\{\phi + \tan^{-1}(\sigma'_2/\sigma'_1)\} \\ &= GE\sigma \sin\{\phi + \tan^{-1}(\sigma_2/\sigma_1)\}. \end{aligned} \tag{39}$$

This is the characteristic velocity of the ionization in a uniform medium, and is to be compared with U_R in equation (34) if we wish to assess the differential motion between the cylinder and the undisturbed medium. Clearly this is greatest when k is small; then the medium moves past the nearly stationary cylinder with the velocity U'_r of equation (39). If E is produced by a wind (of speed $+E/H$ in the direction $\phi = -\frac{1}{2}\pi$) the velocity of the latter must be superposed on both U'_r and U_R. In this event, for all regions of the ionosphere above the D region the dense cylinder will move with nearly the velocity of the wind, while (for the F regions) the ionization of the medium will remain relatively stationary. In the lower regions (where $\tan\alpha_e \to 0$), both the cylinder (however ionized) and the medium will move with substantially the velocity of the wind. In the upper E region the velocity of the ionization in the medium may differ considerably in direction and magnitude from that of the wind.

Equations (19) to (29) are somewhat complicated algebraically, and it is not easy to see their physical significance. The complication is due to the superposition of the current and ionization velocity in the uniform medium upon the perturbations due to the presence of the cylinder. Since the currents and drift velocities are linear functions of the field, we may, without loss of generality, consider the perturbations of these quantities separately. Writing i for the *perturbation* of j, and u for that of U, then

$$i_r = \sigma_1\{(1-k)\,E\cos\phi - \partial S/\partial r\} - \sigma_2\{(1-k)\,E\sin\phi + \partial S/r\,\partial\phi\}, \tag{40}$$

$$i_\phi = -\sigma_1\{(1-k)\,E\sin\phi + \partial S/r\,\partial\phi\} - \sigma_2\{(1-k)\,E\cos\phi - \partial S/\partial r\}, \tag{41}$$

$$i'_r = -k(\sigma_1\,\partial S'/\partial r + \sigma_2\,\partial S'/r\,\partial\phi), \tag{42}$$

$$i'_\phi = -k(\sigma_1\,\partial S'/r\,\partial\phi - \sigma_2\,\partial S'/\partial r). \tag{43}$$

(It will be noted that i is perturbed by the changed conductivity of the cylinder, as well as by the polarization (S) field set up, while i' is perturbed only by the field due to S'.)

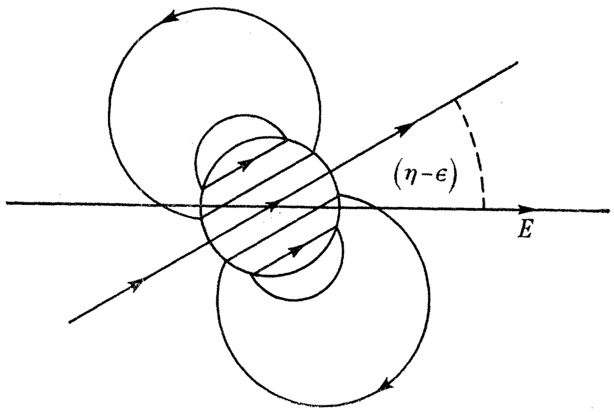

FIGURE 2. Stream-lines of perturbation current i due to cylinder (circular) of ionization density $\frac{3}{2}$ times density of surrounding medium. Conditions otherwise similar to those of figure 1.

The solution of these equations by the methods already used gives (writing $\epsilon = \tan^{-1}(\sigma_2/\sigma_1)$)

$$i_r = \frac{k(1-k)\,\sigma^2 E}{a}\cos(\phi+\eta-\epsilon), \tag{44}$$

$$i_\phi = \frac{-k(1-k)\,\sigma^2 E}{a}\sin(\phi+\eta-\epsilon), \tag{45}$$

$$i'_r = k(1-k)\,\sigma^2\frac{E}{a}\frac{R^2}{r^2}\cos(\phi+\eta-\epsilon), \tag{46}$$

$$i'_\phi = k(1-k)\,\sigma^2\frac{E}{a}\frac{R^2}{r^2}\sin(\phi+\eta-\epsilon). \tag{47}$$

The pattern of current perturbation takes the form typical of a dipole source. The axis of the dipole is inclined, however (for $\sigma_2 \neq 0$), to the direction of current flow in the unperturbed region. The perturbation current stream-lines are illustrated in figure 2, for the conditions of figure 1, but with $k = \frac{2}{3}$. It is clear that when Hall conductivity is appreciable the current stream lines will be both concentrated and twisted or 'kinked' in the vicinity of the cylinder. However, the current flows without divergence both inside and outside the cylinder, and since $i_R = i'_R$ there is no growth of charge on the cylinder walls.

The formulation of u requires care. U is independent of changes in N, since $G\sigma$ varies as σ/σ_0. Hence the u's are proportional to the current perturbations produced by the polarization field alone; the current perturbations in (44) and (45) due directly to differing conductivities ($k \neq 1$) must be excluded. Then

$$u_r = -G(1-k)\,\sigma^2 \frac{E}{a} \sin(\phi+\eta+\epsilon), \tag{48}$$

$$u_\phi = -G(1-k)\,\sigma^2 \frac{E}{a} \cos(\phi+\eta+\epsilon), \tag{49}$$

$$u'_r = -G(1-k)\,\sigma^2 \frac{E}{a}\frac{R^2}{r^2} \sin(\phi+\eta-\epsilon), \tag{50}$$

$$u'_\phi = +G(1-k)\,\sigma^2 \frac{E}{a}\frac{R^2}{r^2} \cos(\phi+\eta-\epsilon). \tag{51}$$

When $\epsilon \neq 0$ ($\sigma_2 \neq 0$), these equations describe a flow pattern not encountered in classical hydrodynamics. Certainly, there is no divergence of fluid either inside or outside the cylinder, but on its wall $u_R \neq u'_R$, and the cylinder, initially moving as a solid with velocity u_R (max) will soon become distorted due to accretion on one side surface and depletion on the other. The stream lines of the perturbed ionization flow are shown in figure 3, for the conditions which applied in figure 2. Their skewness, and the unstable conditions on the cylinder boundary are readily apparent.

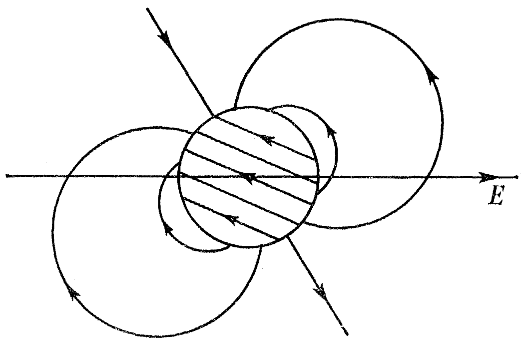

FIGURE 3. Stream-lines of perturbation drift-velocity u of (neutral) ionization due to cylinder of enhanced ionization density as in figure 2.

The foregoing conclusions have been reached, by simple analysis, on a basis of classical theory. The only possible alternative procedure appears to be to assume continuity of ionization flow at the boundary of the cylinder. In this event it becomes impossible to equate i_R and i'_R so that heavy accumulations of free charges would appear on the walls of the cylinder, positive on one side, negative on the other, so producing electrodynamic instability. Since the electrostatic forces opposing accumulations of free (unneutralized) charges are much larger than the diffusion forces opposing concentrations of neutral ionization there seems little doubt that our initial assumption ($i_R = i'_R$) is substantially correct, and that the instability will first manifest itself by establishing pronounced inhomogeneities of neutral ionization. It follows that electric currents in a medium like the E region of the ionosphere may be substantially stable electrodynamically, but the concomitant drift of (neutral) ionization is hydrodynamically unstable, and must necessarily produce marked inhomogeneities in electron (and ion) densities and gradients. It should be remarked, however, that for the ionosphere these inhomogeneities will be confined almost completely to the ionization, and will not extend to the neutral gas (air) containing it. At the pressures existing in the E region the reaction of movement of ionization upon the air is negligible; in the F_2 region it is appreciable, but here $\sigma_2 \ll \sigma_1$, and the flow of ionization under the influence of electric forces is relatively stable.

4. Horizontal and vertical ionization drift in a thin plane sheet with inclined magnetic field

The discussion above has been concerned with electric and magnetic fields mutually at right angles. In the ionosphere, which may be considered as a thin spherical conducting sheet, the magnetic field is inclined to the surface of the sheet. It seems safe to assume (see I) that (vertical) currents perpendicular to the sheet are negligible, their flow being

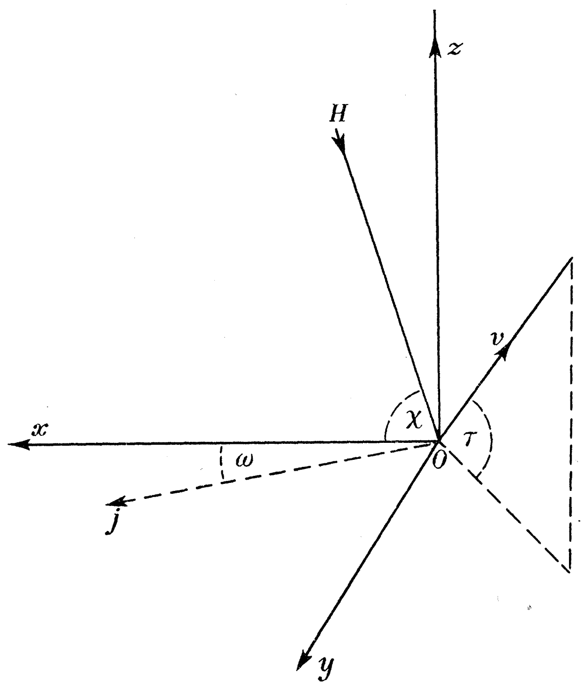

Figure 4. Ionization drift-velocity in ionosphere. The magnetic field H lies in z, x-plane, inclined at angle χ to the horizontal. The current density j is horizontal, at angle ω to x (south) axis. The drift velocity v is inclined at angle τ to the horizontal plane.

prevented by a vertical polarization field. Thus the problem of current flow becomes essentially two-dimensional, the effective conductivities σ_{xx}, σ_{yy}, and σ_{xy} (I) implicitly taking account of the concomitant vertical field. In (I) the currents in such a sheet were studied. It is here proposed to study the drift of the (neutral) ionization, using the principles developed in § 2 above. This drift will be perpendicular to both j and H, and will necessarily have both horizontal and vertical components; although vertical *currents* are prevented by polarization, there is nothing to prevent vertical drift of neutral ionization save the comparatively weak forces of diffusive equilibrium.

The conditions examined are shown in figure 4. Here the x and y axes point south and east (magnetic) respectively, z being vertically upwards. The magnetic field lies in the z, x-plane, and is inclined at the angle χ to the horizontal. Electric fields E_x, E_y are postulated, producing a current density j, lying wholly in the xy plane, at an inclination ω to the

x-axis. The ionization drift velocity v is along OB, which is perpendicular to the plane containing j and H. It is desired to find the vertical and horizontal components of v, viz. $v \sin \tau$ and $v \cos \tau$.

The current densities are

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y, \tag{52}$$

$$j_y = \sigma_{yy} E_y - \sigma_{xy} E_x, \tag{53}$$

$$j^2 = (\sigma_{xx}^2 + \sigma_{xy}^2) E_x^2 + (\sigma_{yy}^2 + \sigma_{xy}^2) E_y^2 + 2\sigma_{xy}(\sigma_{xx} - \sigma_{yy}) E_x E_y, \tag{54}$$

while

$$\tan \omega = j_y/j_x = (\sigma_{yy} E_y - \sigma_{xy} E_x)/(\sigma_{xx} E_x + \sigma_{xy} E_y). \tag{55}$$

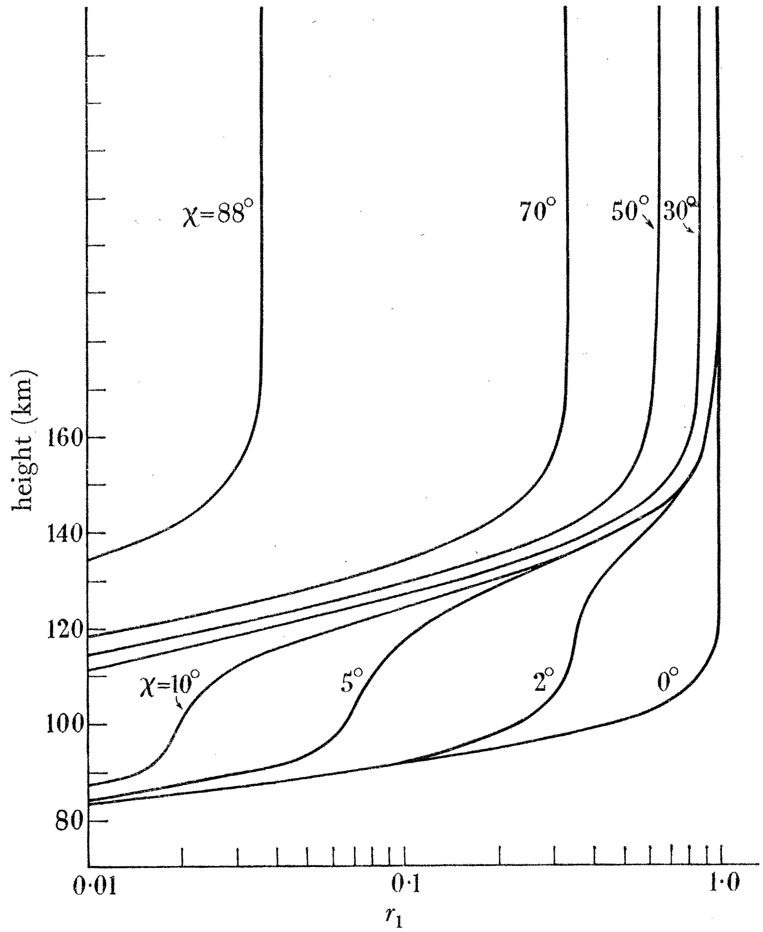

FIGURE 5. Height variation of vertical drift mobility of ionization in the ionosphere at various latitudes (i.e. at various magnetic dip angles χ), for eastward electric fields. The number plotted is r_1, such that the vertical upward drift velocity is $r_1 E_y/H$.

Solving the spherical triangle formed by the intersections of the j, H and x directions from 0 with a sphere of unit radius, it is readily found that Ω, the angle between j and H is given by

$$\sin^2 \Omega = 1 - \cos^2 \omega \cos^2 \chi. \tag{56}$$

Hence

$$v = Gj \sin \Omega$$

$$= G\{(\sigma_{xx}^2 \sin^2 \chi + \sigma_{xy}^2) E_x^2 + (\sigma_{yy}^2 + \sigma_{xy}^2 \sin^2 \chi) E_y^2 + 2\sigma_{xy}(\sigma_{xx} \sin^2 \chi - \sigma_{yy}) E_x E_y\}^{\frac{1}{2}}$$

$$= G(j^2 \sin^2 \chi + j_y^2 \cos^2 \chi)^{\frac{1}{2}}. \tag{57}$$

The angle τ is given by $$\tan\tau = \cot\chi \sin\omega. \tag{58}$$

It readily follows that the vertical and horizontal drift velocities are, respectively,

$$v \sin\tau = Gj_y \cos\chi, \tag{59}$$

$$v \cos\tau = Gj \sin\chi. \tag{60}$$

Thus in a given region of the ionosphere the vertical drift of ionization depends entirely on the *east-west current in that region*, while the horizontal drift depends on the total current. In the E region this means that vertical drift will be greatest when the horizontal electric field is substantially (but not exactly) in the north-south direction. On the other hand, in

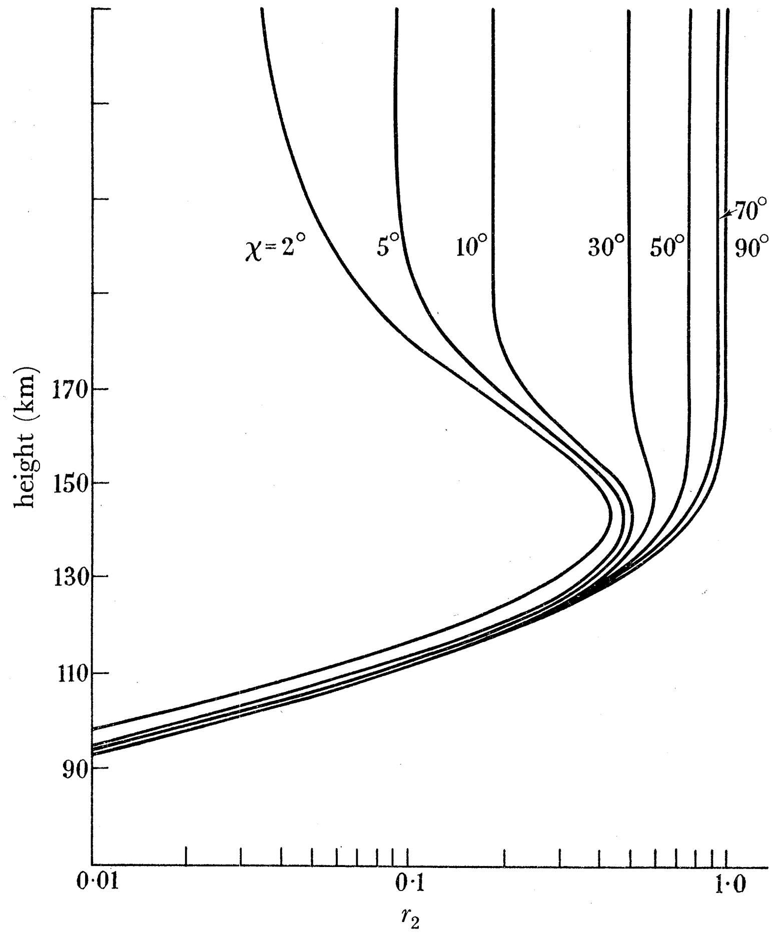

FIGURE 6. Height variation of horizontal drift mobility of ionization in the ionosphere, at various latitudes (or dip angles χ) for eastward electric fields. The number plotted is r_2, such that the horizontal drift velocity is $r_2 E_y/H$.

the F_2 region, vertical drift will be greatest when the field is substantially east-west. In both hemispheres, in a given region of the ionosphere, the drift will have its greatest upward velocity when the local current density is greatest eastwards. It seems certain that, at the ionospheric levels where flow the currents mainly responsible for the solar and lunar magnetic variations, the daily oscillations of these regions at low latitudes must be in phase opposition to those at high latitudes.*

* This conclusion was previously reached (Martyn 1947) by more primitive argument.

Equations (59) and (60) express the vertical and horizontal drifts concisely, and expose the underlying physical cause. For many practical purposes it is preferable to express these drifts in terms of E_x and E_y. For example, if these are 'dynamo fields' it is necessary, for the case of horizontal drift, to superpose the wind velocity upon the drift velocity in order to obtain the resulting motion of the ionization. These drifts are illustrated in figures 5 to 8, for ionospheric levels, and for a range of χ representative of all latitudes. Figure 5 shows the height and latitude variation of a quantity r_1 such that

$$r_1 E_y/H = v \sin \tau \quad (E_x = 0).$$

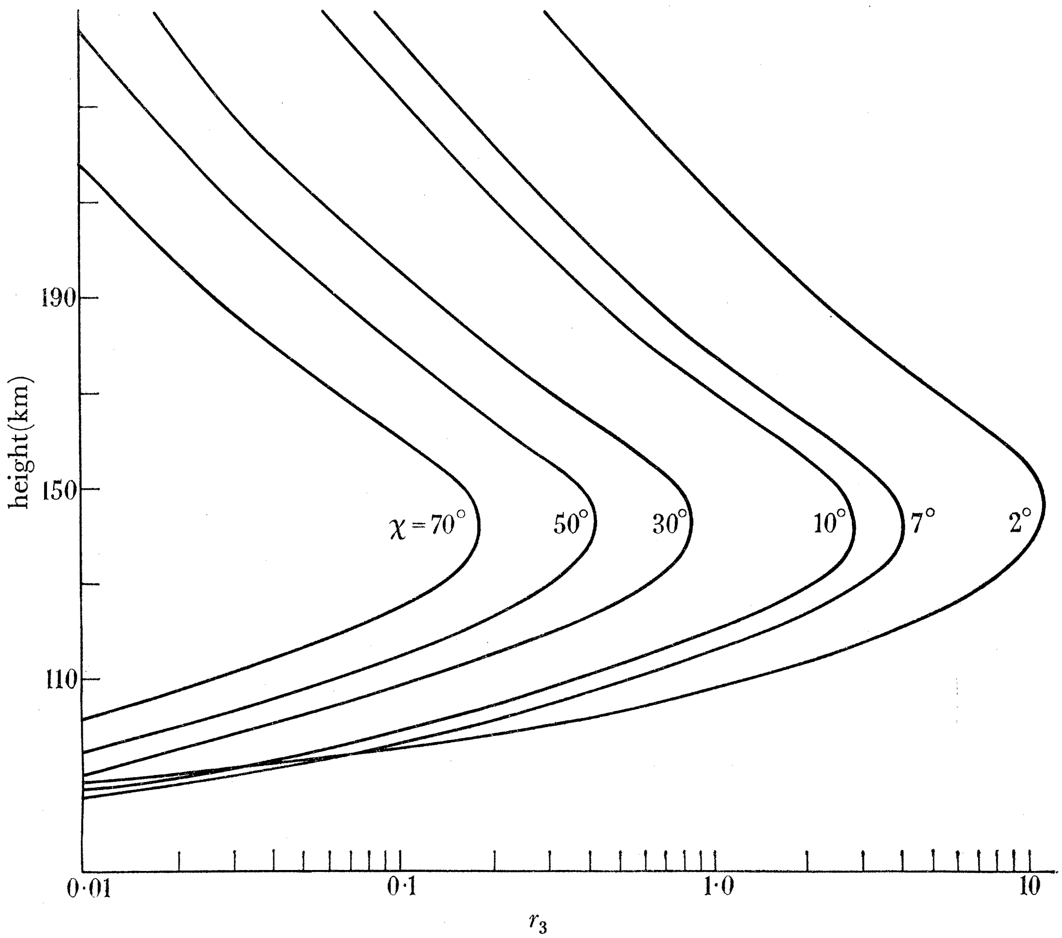

FIGURE 7. Height variation of vertical drift mobility of ionization in the ionosphere, at various latitudes (i.e. magnetic dip angles), for southward electric fields. The number plotted is r_3 such that the vertical (upward) drift velocity is $-r_3 E_x/H$.

Figures 6, 7 and 8 show r_2, r_3, r_4 such that

$$r_2 E_y/H = v \cos \tau \quad (E_x = 0),$$
$$-r_3 E_x/H = v \sin \tau \quad (E_y = 0),$$
$$-r_4 E_x/H = v \cos \tau \quad (E_y = 0).$$

It will be seen from figure 5 that the vertical drift due to an east-west field is small in the E and lower regions, save very close to the equator; at moderately high latitudes it approaches its maximum value at a level somewhat below the F_1 region. The vertical drift due to a

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north-south field (figure 7) is larger than for east-west fields at E region levels, and is a maximum at about 150 km. This drift (r_3) increases at first as the equator is approached, and is a maximum when $\tan^2\chi = \sigma_1/\sigma_0$; at lower latitudes it decreases rapidly, disappearing at the equator. It becomes very small in the F region at all latitudes.

Figure 6 shows that horizontal drift due to east-west fields is small below about 110 km. It reaches its maximum value at about 140 km; above this height it is relatively constant at moderate and high latitudes, but decreases rapidly near the equator.

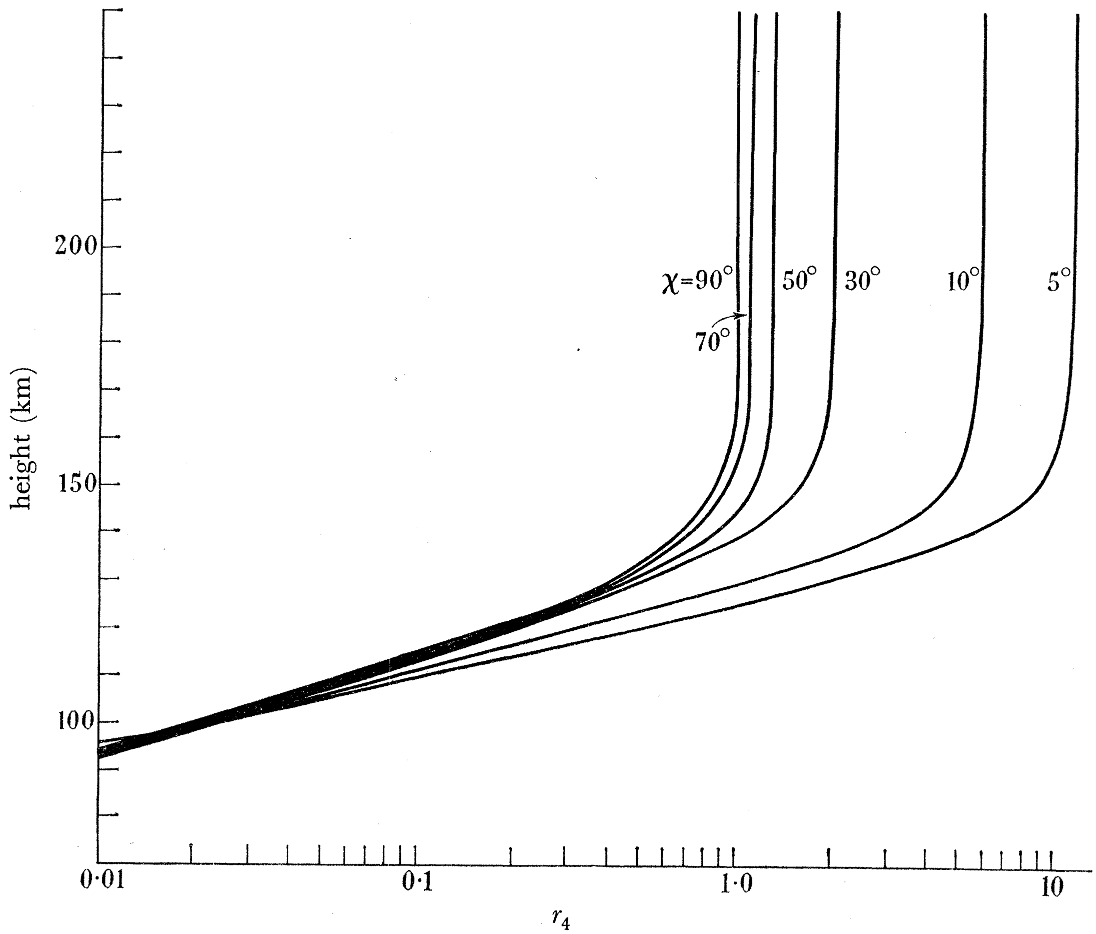

FIGURE 8. Height variation of horizontal drift mobility of ionization in the ionosphere, at various latitudes (i.e. magnetic dip angles) for southward electric fields. The number plotted is r_4 such that the horizontal drift velocity is $r_4 E_x/H$.

The horizontal drift due to north-south fields is appreciable above 110 km, and approaches its full value at about 150 km, remaining constant at greater heights; it disappears close to the equator.

The following general conclusions, of immediate importance for the experimental study of ionospheric 'winds', may be drawn, remembering that in all such cases the wind vector has been added to the 'dynamo' drift:

(1) In the lower E region, and below, ionization will move horizontally with substantially local wind velocity.

(2) In the upper E region the horizontal motion of ionization may differ substantially in magnitude and direction from that of the local wind.

(3) In the F regions ionization cannot be moved by winds transverse to the earth's magnetic field. Thus an east-west wind here can produce no appreciable movement of ionization. A local north-south wind causes the F region ionization to move along the direction of the earth's field, with (very nearly) the velocity of the wind component in that direction ($c_0 \cos \chi$); then the north-south horizontal drift is $c_0 \cos^2 \chi$. High east-west drift velocities can be produced in the F regions only by north-south electric fields communicated from elsewhere.

General conclusions applicable to the study of vertical drifts of the ionosphere are:

(1) Vertical velocities due to either winds or fields are small below 100 km, save in the immediate vicinity of the magnetic equator; near this equator east-west fields can produce notable vertical drifts at heights of 90 km and upwards.

(2) In the F regions notable vertical drift is produced by east-west fields, and/or by local north-south winds; in the latter case the wind simply blows the ionization along the direction of the earth's field, the vertical component of drift being $c_0 \sin \chi \cos \chi$. North-south fields or east-west winds produce no appreciable vertical drift.

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